9. Thermal Structure of the Solar System

Planetary Temperatures

We can calculate the temperatures that we would expect the planets to have using some simple heat balance equations.

The total energy received over the whole of a planet is given by the area it presents to the Sun times the radiation per square meter at that point. The radiation power per square meter at the earth is given by S, the solar constant (around 1360 W m^{-2}), and we know S falls off according to the inverse square law. Hence energy received by planet:

$$= (S/a^2)[pi]R^2$$

where a is the distance from the sun in AU.

Effective Temperature, T_e:

If a body and its atmosphere is at a uniform temperature T_e, then it will emit as a black body radiator of effective temperature T_e.

Rate of emission = $4[pi]r^2[sigma]T_e^4$

where [sigma] is Stefan's Constant.

If we define the albedo as (Energy reflected)/(Total energy received), and call this A, then the total energy absorbed is given by:

$$\text{Total Energy Absorbed} = [pi]R^2S\ (1-A)/a^2$$

where a = a_p/a_e, and a_p is the distance of the B > p is the distance of the planet from the sun in kilometers (and a_e the same for the earth.) Therefore we can say

$$T_e^4 = (1/4)(S/a^2)(1-A)/[sigma]$$

We can express S in terms of the black body temperature of the Sun T_s:

$$4[pi]a_e^2S = 4[pi]R_s^2[sigma]T_s^4$$

Here R_s is the radius of the Sun.

So:

$$T_e^4 = R_s^2T_s^4(1-A)/4a_e^2a^2$$
$$= R_s^2T_s^4(1-A)/4a_p^2$$

or:

$$T_e = (1/k)T_s(1-A)^{1/4}(R_s/a_p)^{1/2}$$

k here is $=2^{1/2}$ in the case of a uniform radiator (planet that spins fast or has an atmosphere with fast winds. Alternatively k might be $2^{1/4}$ if it radiates uniformly from one hemisphere only (this might be a slow rotator, or a non-rotator with good surface heat conduction). k is 1 if local equilibrium is maintained at a subsolar point (non-rotator).

The average surface temperature of the earth is higher than T_e calculated like this by about 45K on account of the greenhouse effect. This formula of course also applies to bodies other than the planets -to the satellites of the planets alites of the planets and artificial satellites as well.

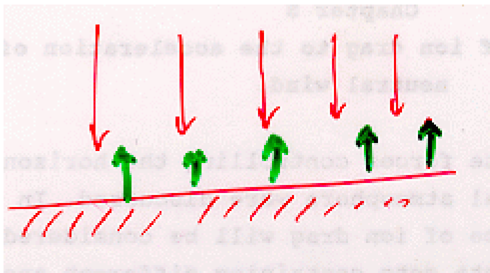
	Mercury	Venus	Earth	Mars	Jupiter	Saturn	Uranus	Neptune	Pluto	Moon
Albedo	0.16	0.65	0.37	0.15	0.52	0.47	0.5	0.41	0.6	0.174
T_e spherical	428	253	249	218	102	77	53	45	35	
T_e hemi-spherical	509	301	296	259	121	92	64	53	42	
T_e sub-solar	606	358	352	308	144	109	76	63	50	373

The moon's observed subsolar temperature is 370 K, falling to 203 at sunrise.

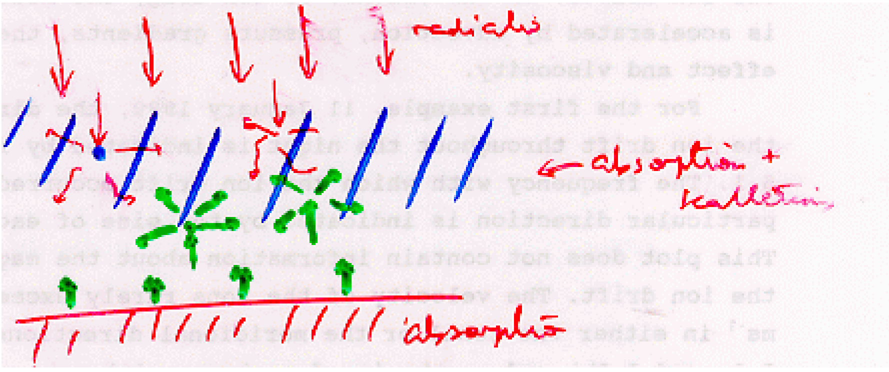
Measurements of Mercury show 600-700 K subsolar (i.r), but 400 K average over the disk (microwave) which requires either a cross between spherical and hemi- spherical balance, or a change in albedo. Venus before space probes was shown to have a 225 K average temperature with 240 K centrally, suggesting an even distribution by winds or rotation (now known to be winds, of course), but microwave measurements gave a temperature of 600 K. It is now known this is because the microwave radiation comes from the surface (see below).

Energy Budget with an atmosphere

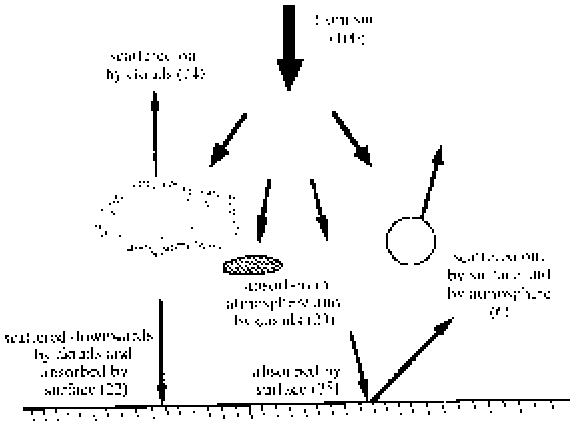
When you have to take into account a "real" atmosphere the analysis becomes more complicated. With no atmosphere we get some radiation reflected at the same wavelength as the input and, some absorbed which heats the surface. The heat in turn leace. The heat in turn leads to re-radiation at a wavelength greater than the input (infra red usually).



With an atmosphere we now get some radiation absorbed and re-radiated (often multiple times) both before reaching the surface and after reflection/re-radiation. Thus, we have absorption and scatter in the atmosphere and absorption by the surface complicating the simple picture.



The earth's atmosphere is transparent in the regions where most of the sun's energy falls. A third is reflected at the same wavelength (albedo), and 2/3 reflected at IRwavelengths (2/9 from the surface, 4/9 transported by radiative transfer or convection. (These proportions will depend on the cloud cover too, of course.)



For the earth all wavelengths less than 300 nm are absorbed and reflected. (This is less than 1% of the total solar radiation.) That radiation reflected from the surface is not much affected by the atmosphere, but that which is absorbed and re-radiated is radiated into space at some wavelengths but not at others. In particular, the tri-atomic molecules H_2O and CO_2 absorb much of it.

The process of absorption and re-radiation upwards continues until the density falls so far that there are insufficient triatomic molecules to absotriatomic molecules to absorb the radiation. Up to that point we have a mix of radiative transfer and convective transfer. For the radiative transfer the net flux = F_1 - F_2, and is a constant with height, where F_1 is the upward flux and F_2 is the downward flux.

The upper level for both radiative and convective transfer is the tropopause since radiative transfer maintains the temperature gradient that enables convective transfer to be maintained. Where F_2=0 te temperature stops decreasing with height and so convective transfer stops.

(This is not wholly true - other factors come into play, but we expect the levels where these two heat transfer processes stop to be close.)

Dry Adiabatic Lapse Rate

The "lapse rate" of an atmosphere is the rate at which temperature falls off with height (that is a positive lapse rate is a negative change of temperature with height).

The adiabatic lapse rate is that maximum change of temperature with height which is stable to vertical movement: this is usually given as the "dry adiabatic lapse rate" in the case of the earth where there is a significant proportion of water vapour in the atmosphere because the presence of water (with its high latent heat) can make a significant difference to the stability. To explain the adiabatic lapse rate we must look at the stability of an atmosphere. ability of an atmosphere. This will be stable against convective overturning as long as its temperature rises with height. However, even a small fall in temperature with height can be stable because denser air from below rising will expand and thus cool adiabatically: if it cools faster than the temperature drops with height then it will become denser than its surroundings and fall back again (or settle in to the height where it hydrostatically balances) rather than continuing to rise and cause a convective instability. Another way of looking at this is to note that the lower air, being hotter, will expand. It will thus, if this were a uniform gas, become less dense (i.e. lighter) than its surroundings. Again, if it were a uniform gas, it would try to rise. However, the pressure falls off with height (see below - hydrostatic equation) due to the fall off of the super-incumbent mass of atmosphere with height. Provided this pressure fall off gives a rate of density fall-off with height which is greater than the density fall due to the temperature gradient, then the hotter air below will not become lighter than the cooler air above it, and so not rise. The balancing point for the temperature gradient - that is the gradient which is just balanced by the hydrostatic density gradient - is the adiabatic lapse rate.
We can calculate what this lapse rate is using the hydrostatic equation:

$$dp/dz = -g[ro]$$

where [ro] is the density, p pressure, z height and g the acceleration due to gravity. We also have the 1st Law of Thermodynamics:

$$C_v dT = - P dV$$

so

$$dT/dz = - (1/C_v) P dV/dz$$

Perfect Gas Law:

$$pV=RT$$

so

$$Vdp + pdV = RdT$$

so

$$dV = RdT/p - RTdp/p^2$$

Now, since $C_p = C_v + R$

we have

$$C_p dT = C_v dT + RdT = -pdV + RdT$$
$$= Vdp = -dp/[ro]$$

Therefore

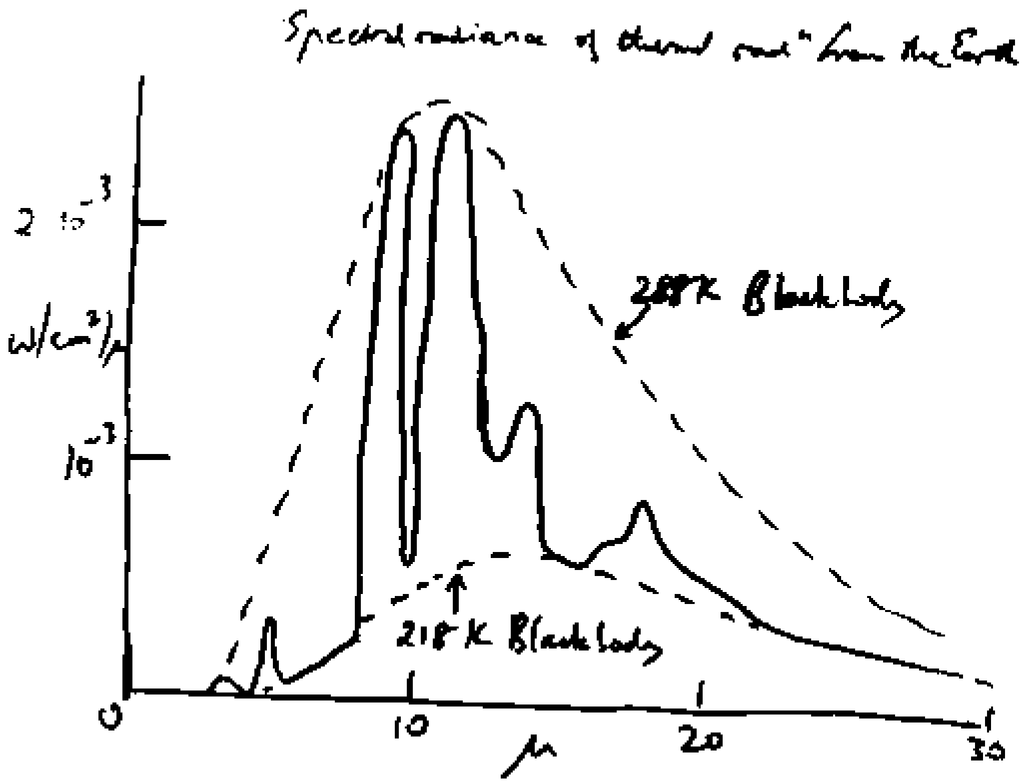
$$dT/dz = (1/C_p)(1/[ro])dp/dz = -g/C_p$$

-dT/dz is the lapse rate, of course.

The troposphere has a lapse rate of 9.6degrees/km rather than the 6.5 needed for convective instability in a static atmosphere so it is often unstable and has convective motions transporting the heat. However, the release of heat from water vapour as its pressure and density changes has a large effect on the heat content of a parcel of air and so the picture is a lot more complex than the "dry air" argument would suggest.

Temperatures fall to < 273 K at the tropopause and so this acts as a water vapour trap. CO_2 is not held here, though, and is fully mixed up to the 100km level (though it is not optically thick here and some radiation gets through).

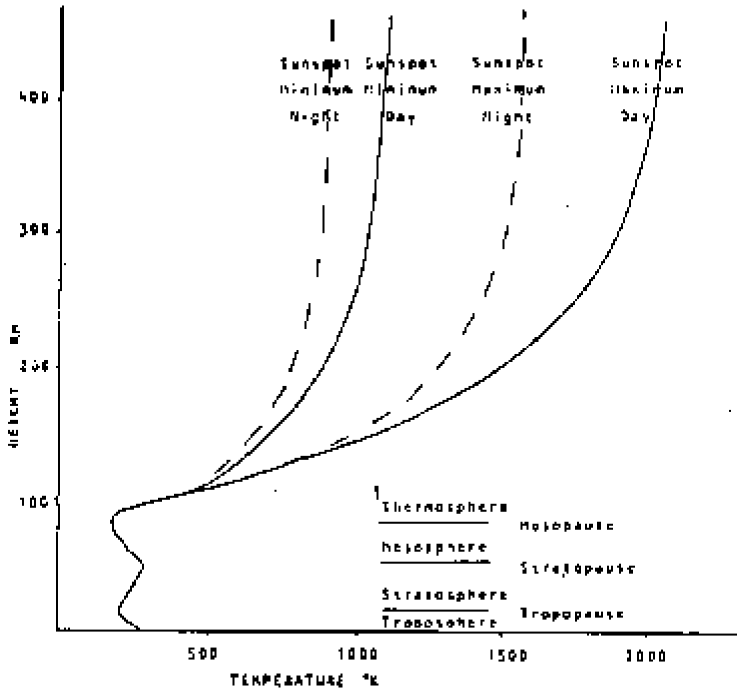
Spectral Radiance of thermal radiation from the earth



The top curve is 37-93.gif">

The top curve is the surface infra-red radiation (- - -) filtered through the atmospheric transmission characteristics. Below 10 microns the water vapour is the main absorber, above 12 microns it is CO_2 and H_2O. At 9.6 microns there is an ozone (O_3) absorption band. The temperature at the extremes of wavelength is rather that of the emitting gas at more or less the tropopause (see below) - 218K. The simple temperature balance equation above gave an expected effective temperature for the earth of 246K (T_e spherical). What we actually see is 218K and a modulated 288K, averaging out to what you would get from a 246K Planck curve.

Temperature profile of the earth's atmosphere



Earth: Temperature vs Height graph

Note that the structure is defined as a number of layers (-spheres) separated by -pauses which are defined as the inflection points - thus the tropopause defines the top of the troposphere and is where the temperature gradient turns over from negative to positive. The troposphere is heated mainly by the ground, which absorbs solar radiation and re-emits it in the infra-red. Radiative transfer and convection distributes this heat through the lower atmosphere - see the notes above on lapse rate. The temperature of the troposphere drops off at a rate of approximately 1 K every 165 m. The tropopause is at a temperature of about 210-220 K, so it acts 210-220 K, so it acts as a water "trap" and water vapour cannot generally pass through this to the atmosphere above. The height of the tropopause varies with latitude, being 11km or so at the equator but as little as 7 km often at the poles. Because it is so much higher (and the temperature falls off faster with height) at the equator, the equatorial tropopause is much colder than the polar tropopause.

The stratosphere above this is the region with a positive temperature gradient: this is due to heating from the Ozone which, though it forms only a tiny fraction of the air by volume (never more than $1:10^5$), is an important absorber of the ultra-violet wavelengths of solar radiation which penetrate down to these altitudes. The Ozone density peaks at about 30 km (though its absolute density there is still only 6-10 times that at sea level), but it stretches up to 80 km, and the maximum temperature is at around 50 km as this is where the majority of the absorption of incoming radiation occurs. Above 50 km the density of ozone drops off faster than the increase in incoming radiation can compensate for. The ozone is the main absorber because of its large absorption cross section, but it, together with CO_2 is also a major source of heat loss by emission of long wavelength IR radiation. In the stratosphere the heat input outweighs the emission.

The inflection point at 50 km is called the stratopause, anthe stratopause, and the region above this is the mesosphere. This is often called the "ignorosphere" because it is so difficult to get detailed measurements from here, though a sustained effort in recent years is now beginning to pay off. There are no in-situ heat sources in the mesosphere of note but it loses heat by emission from CO_2, O_3 and (higher up) NO, so it is a cold region, and drops in temperature to the mesopause at about 85 km which is the coldest place on Earth. Curiously it is coldest in summer due to the counterintuitive circulation pattern here. There are a number of theories to explain this (ranging from chemical effects to adiabatic cooling of ascending air driven by large-scale horizontal circulation). The mesopause acts as a sink for the heat generated in the thermosphere above it.

The thermosphere is heated mainly by absorption of EUV and XUV radiation. Up to 105 km the atmosphere is turbulently mixed and has a uniform composition with altitude. At 105 km (the turbopause) the eddy diffusion (turbulent mixing) gives way to molecular diffusion (laminar flow), which means in essence that the different species act independently and distribute themselves with height as if the other constituents did not exist. Each species is therefore in hydrostatic equilibrium with its behaviour governed by the Perfect Gas Law p=nkt (or PV=RT) and the pressure fall-off with height given by the reductgiven by the reduction in the superincumbent mass of that species – dP = nmg dz (P = pressure, n = no. density, m is the molecular mass of a particle, k is Boltzmann's Constant, T = temperature and g = acceleration due to gravity).

This leads to the definition of scale height, H, which is the height over which the density falls off by a factor 1/e:

$$\frac{dp}{p} = \frac{mg}{kT}\,dz \qquad \therefore\ p = p_0\, e^{\int \frac{mg}{kT}\,dz}$$

$$= p_0\, e^{\int \frac{dz}{H}}$$

where H = kT/mg (or RT/Mg). In the lower atmosphere this is around 8.3km, but above 105km the temperature climbs steeply until it reaches the asymptote shown above, and the molecular mass m of each species controls the relative fall off with height. The solar XUV dissociates the O_2 into atomic oxygen, and since this has a lower atomic mass than the molecular mass of O_2 and N_2, it falls off slower with height until it dominates above around 150 km. Above 105 km H rises quickly to be 50-75 km. The mean molecular mass is 29 up to 105 km, 25 at 200 km and 18 at 400 km. (Above this is drops even more as Hydrogen and Helium become the dominant species.)

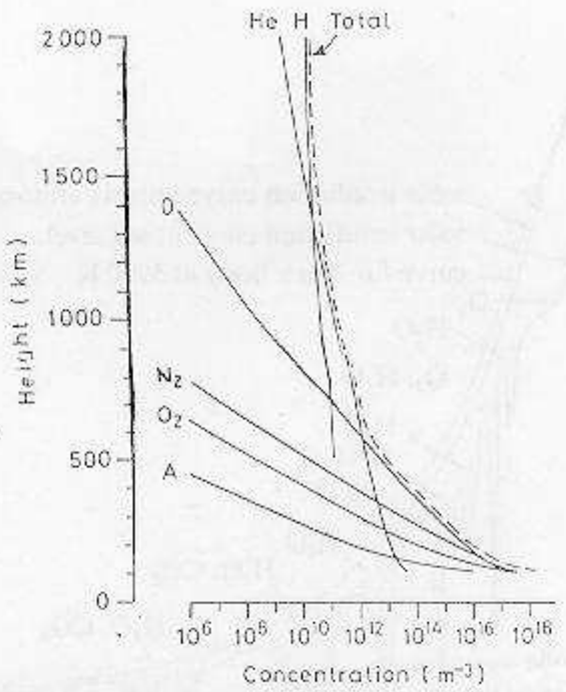
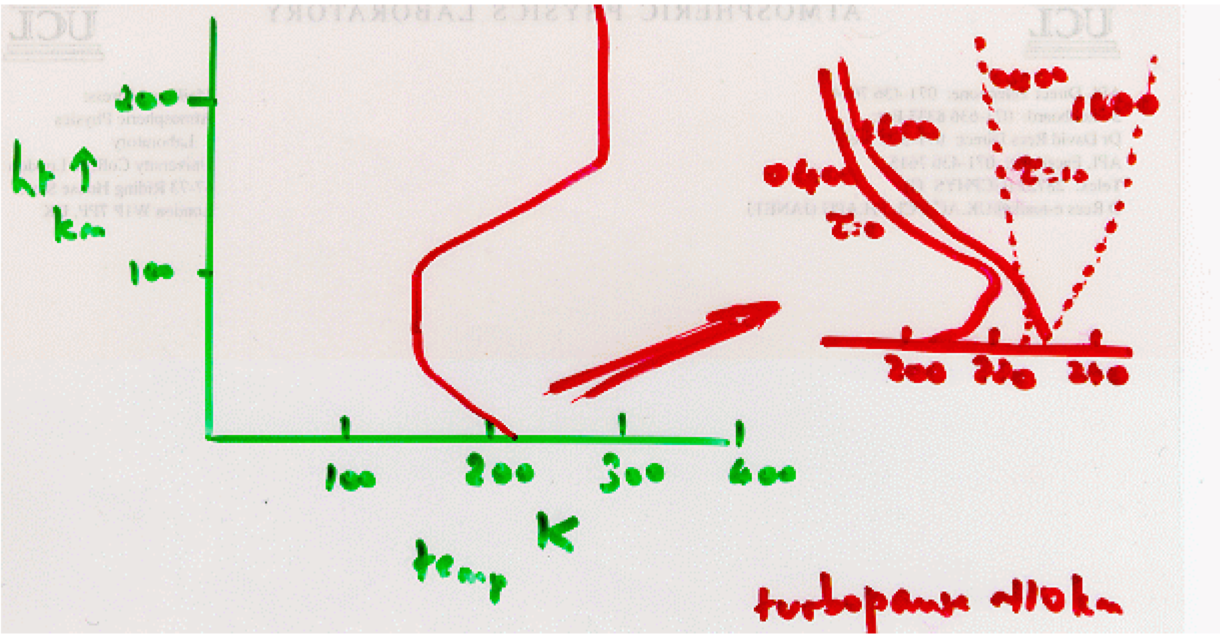


Figure 4.3 Distribution of major gases in the thermosphere for an exospheric temperature of 800°K. (Data from COSPAR International Reference Atmosphere, 1972).

In the thermosphere the main heat input is the dissociation of molecular oxygen, which creates atomic oxygen carrying the excess energy of the dissociation reaction, and absorption of quanta of radiation by atomic oxygen which are not energetic enough to ionize it. Since O is a poor radiator, the O is a poor radiator, the temperature builds up to 100s K or 1000s K. The main heat loss is by conduction down to the mesopause (though it has very little effect on the temperature there since the density of the thermosphere is so small compared to that of the lower atmosphere). Since the heating is due to EUV and XUV this is very solar cycle dependent. You will see above that the thermosphere has a small day-night variation, but a large solar-cycle variation. This ties in with what we noted above about solar cycle variation of the different wavelengths of solar emissions. The main bulk of solar radiation barely varies at all over a solar cycle, which is why the ground temperature does not vary much. The EUV - and even more so the XUV - however vary by anything up to orders of magnitude (depending on wavelength). All wavelengths below 300 nm are absorbed high in the atmosphere, though this is less than 1% of the total solar flux.

Temperature profile of Mars' atmosphere

There is a day-night variation close to the surface as Carbon Dioxide is such a good emitter, but note the lack of a stratosphere as there is insignificant amounts of ozone to offset the temperature fall off with height. Intermediate levels are still dominated by transfer from the ground and there is little diurnal variation. The lower atmosphere is therefore effectively one big troposphere, heated by the surface, merging with the "thermosrging with the "thermosphere", which is much colder than the Earth's because it contains a large proportion of Carbon Dioxide to quite high altitudes. Carbon Dioxide is a much better radiator than atomic oxygen (the major species in the earth's thermosphere) and so radiates quickly away the heat generated by absorption of EUV and XUV. The Martian turbopause is at around 110 km.

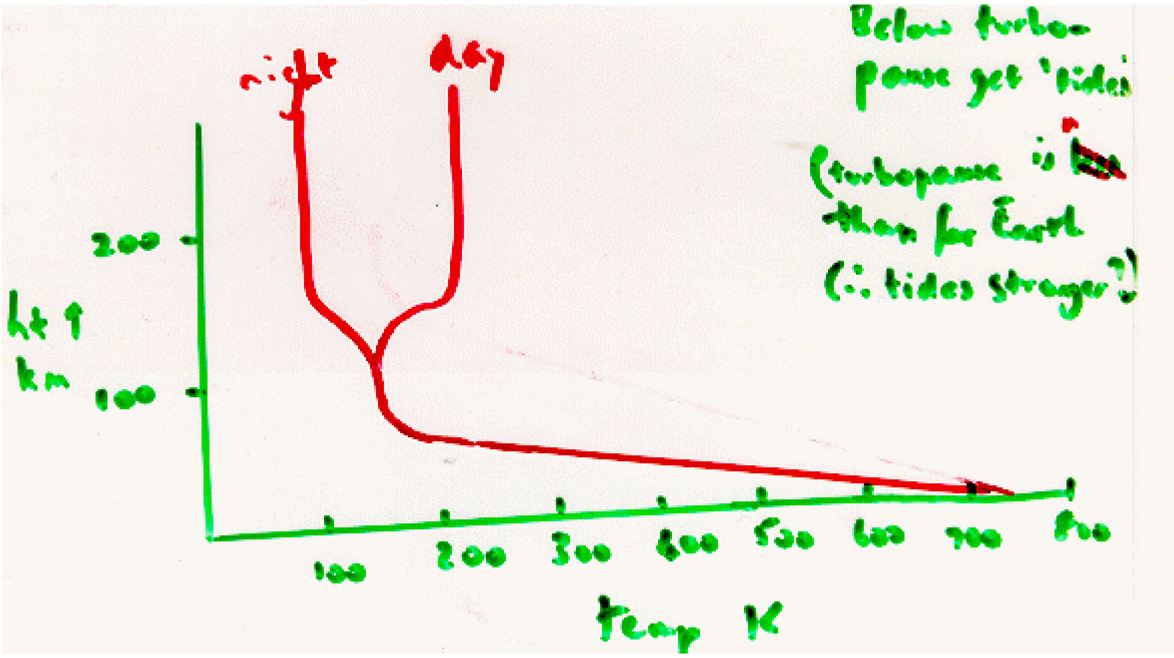


The eccentricity of the Martian orbit means that the southern autumn and winter are longerthan the northern (194/178 days v 143/156). There is also 40% more solar insolation at perihelion. Therefore the southern hemisphere gets shorter but hotter summers and longer, colder winters. (This situation reverses every 25,000 years due to precession of the Martian poles.) The result of this is that the southern polar cap varies more in size, but although both caps have a seasonal CO_2 ice component of a few cm thickness, only the southern cap has Carbon Dioxide in the residual cap in southern summer. The south polar cap varies more in size. The northern residual cap is about 1000 km diameter, mainly H_2O ice at around -68 degrees C, while the southern residual cap is about 350 km diameter and a mixture of water and Carbon Dioxide ice at around -113 degrees C.

There is such a fast melting of the southern cap at the onset of summer that global sandstorms are created, fed by torms are created, fed by the temperature gradient from pole to the defrosted and uncovered ground at mid-latitudes. Therefore the northern polar cap forms with a greater dust content (the 'hood' of polar cloud, which obscures it until spring, when we see the cap retreating at a rate of 20 km/day). The dark and light variation of the Martian surface is largely due to the dust distribution, dark regions being largely free of wind-blown dust, and the lighter areas having a significant covering. The difference between northern and southern summers is so significant in terms of the amount of melting of the caps that there is a distinct seasonal variation of atmospheric pressure due to the differing amounts of CO_2 released.

Because of the lower Martian gravity N, O and C can escape - presumably the reason why the atmosphere is now so thin. It is assumed there was also considerably more water previously (though we do not know how much is locked up in the surface). Despite the very small amount of water in the atmosphere, the total pressure is so low that water is nearly always near saturation.

Temperature profile of Venus' atmosphere



There is a detectable day-night variation in the Venusian thermosphere, but note it is colder than the Earth's, despite Venus' nearness to the Sun, for the same reason that Mars has a cold thermosphere - it is larghermosphere - it is largely CO_2. The greenhouse effect with the enormous atmospheric pressure of Carbon Dioxide is the reason for the very high surface temperature (7-800 K) - this in turn gives a very large lapse rate as the temperature drops to 150 K by 60-75 km altitude. Again, note that there is no stratosphere or equivalent. The turbopause is at around 150 km height. Below this strong tides are generated, and this may be the reason for turbulent mixing to go so high (i.e. the density at the turbopause is less than at the Earth).

Comparison of minor species in the terrestrial planets' atmospheres

Water condenses at such moderate temperatures that the Earth and Mars' atmospheres contain only a little water. On Earth the water goes mainly into the oceans and the ice sheets. On Mars it is adsorbed onto soil grains and into ice in the northern polar regions. Both Earth and Mars are on average at one third saturation in the atmosphere. Venus is so hot all the water vapour is in the atmosphere, and some even combines with sulphur di- and tri-oxide to give Sulphuric Acid (H_2SO_4) droplets at high altitude.

On all three planets there is some dissociation of H_2O to HO and O:

$$H_2O + h\nu \dashrightarrow HO + O$$

The HO is highly reactive (going to sulphuric acid or destroying ozone).

Atomic Oxygen and Ozone

In the Earth's Ozone

In the Earth's upper atmosphere molecular oxygen is broken down to atomic oxygen:

$$O_2 + h\nu \dashrightarrow O + O$$

followed at lower altitudes by $O + O_2 \rightarrow O_3$

On Venus and Mars you get some ozone production from the CO_2:

$$CO_2 + h\nu \rightarrow CO + O$$
$$O \rightarrow O_2 \text{ or } O_3$$

Ozone on Mars is particularly seen over the winter pole: the fact no OH is seen suggests it is formed from CO_2 not H_2O.

Sulphur Dioxide (SO_2)

Sulphur Dioxide in the Earth's atmosphere reacts to form particles which drop to the ground, though there is some sulphuric acid and NH_3SO_4 low down in the atmosphere. This increases due to volcanic activity and power station outflow. On Venus, the heat prevents its removal and so the SO_2 in the lower atmosphere tends to go to H_2SO_4, and a layer is formed around 50-80 km. On Mars there seems to be no sulphur dioxide to note in the atmosphere at all.

Summary terrestrial planetary atmospheres

planet	g $m\ s^{-1}$	pressure Bar	T (K)	major gases:
Mercury	3.95	10^{-15}	440	He(0.42), Na(0.42), O(0.15)
Venus	8.88	90	730	CO_2 (0.96), N_2 (0.035)
(0.96), N_2 (0.035)				
Earth	9.78	1	288	N_2 (0.77), O_2 (0.21) H_2O (0.01), Ar (0.0093)
Mars	3.73	0.007	218	CO_2 (0.95), N_2 (0.027), Ar (0.016)

(Greenhouse +5 degrees Mars, +35 degrees Earth, +500 degrees Venus)

Planet	Particulates (composition)	Altitude(km)	Optical depth: (tau)
Mercury	none		
Venus	Concentrated H_2SO_4	50-80	c. 25
Earth	conc. H_2SO_4	12-30	.003-0.3 (latter volcanic)
	sulphates,silicates, sea salts,organics	0-12	0.05-3 (spatially variable)
	water (50% cloud cover)	0-12	5
Mars	dust	0-50	0.3 -6 (6=dust storms)
	water ice	0-50	c. 1 (winter pole)
		c. 60	c. 0.001
	CO_2 ice	0-25	c. 1 (winter pole)

Composition of Earth's atmosphere at the ground

Molecule	Mass	Percent of Volume	Concentration per cc:	
Nitrogen	28.02	78.1	2.1×10^{19}	
Oxygen	32.00	20.9	5.6×10^{18}	
Argon	39.96	gon 39.96	0.9	2.5×10^{17}
Carbon Dioxide	44.02	0.03	$8.9\ 10^{15}$	

Neon	20.17	0.002	$4.9\ 10^{14}$
Helium	4.00	0.0005	$1.4\ 10^{14}$
Water	18.02	Variable	

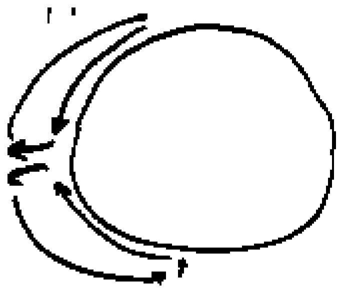
Circulation patterns in terrestrial atmospheres

There are several modes of circulation in terrestrial atmospheres, though the pattern of dominance varies from planet to planet. The major circulation patterns are:

- Hadley Cells
- Large-Scale Eddying (Baroclinic Eddies)
- Large-Scale Eddying (Topographic Effects)
- Condensation Flow
- Thermal Tides

Hadley Cell (Theoretically)

The Hadley Cell circulation pattern is due primarily to differential heating from equator to pole. Thus we expect the air to rise at the equator and fall at the pole, setting up hemispherical circulation cells:

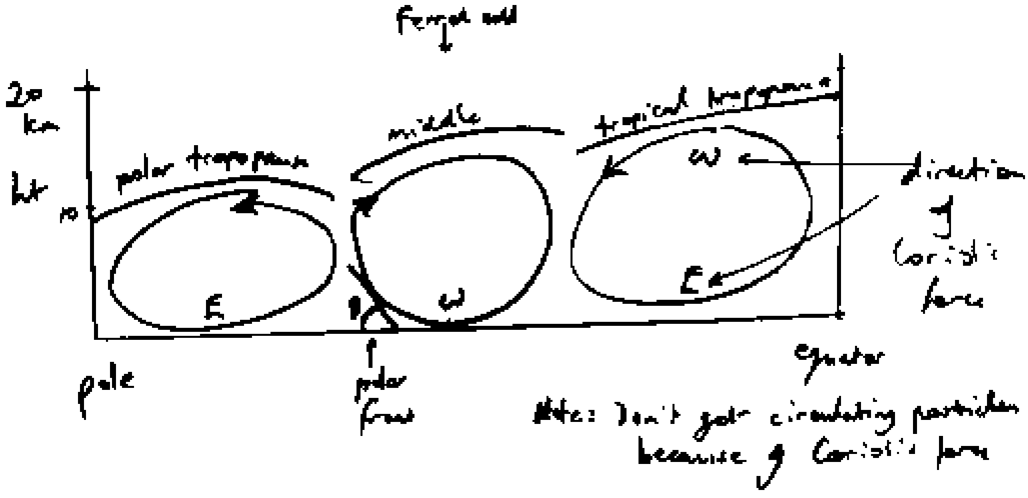


This is, indeed, what the situation on Venus is supposed to be, though rather than one single cell at a given altitude it is believed Venus has a number of Hadley cells packed on top of each other at different altitudes.

On Mars there is believed to be only one cell globally rising in the summer tropics and sinking in the winter subtropics. This is modifed a little by the Coriolis force, but it is believed to be a predictable pattern, making Martian "weather" potentially more predictable than Earth's!

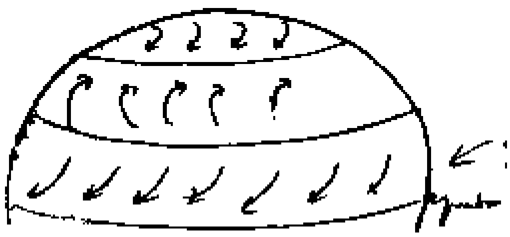
On Earth the pattern is much more affected by Coriolis forces which effectively make a single cell too unstable. The pattern therefore tends to a three-cell one in latitude:

Hadley cells in earth's atmosphere, distorted by Coriolis Forces



The middle cell is called the Ferrel Cell. The air particles do not actually circulate in the longitude plane as shown by the arrows because of the Coriolis forces. The "W" and "E" on the diagram shows the Coriolis forcing, and this leads to the predominant Wind Belts known in each hemisphere, such as the 'trade winds':

The resultant ground-level wind pattern:



Large-Scale Eddying: Baroclinic Eddies

Baroclinic Eddies are the circular patterns we see on weather maps - the cyclones and anticyclones (or "Highs" and "Lows") caused by Coriolis divergence of the flow around a high or low pressure centre. The convergence of two such eddies leads to Jet Streams.

On Venus it is not known if these exist. On Earth there are usually 6-7 pairs of highs and lows circling the globe around the jet streams. They are irregular in formation and duration. On Mars they are thougd duration. On Mars they are thought to only occur in the winter hemisphere and are quasi-periodic (therefore again the Martian weather is more "predictable"?)

Large-Scale Eddying: Topographic Effects

There are a number of effects due to geographic features:

- stationary eddies (e.g due to ocean -> continent temperature or elevation effects)
- lee waves (stationary waves set up in the lee of mountain chains

Such eddies do not propagate horizontally, but can transport heat and momentum vertically. These effects can be large on Mars and Venus due to the size of the topographic features on these planets.

Condensation Flows

As far as we are aware currently this is unique to Mars - the flow due to the condensation and subimation of the Carbon Dioxide in the polar caps effectively transfers the CO_2 from pole to pole, and pole-to-atmosphere and back again. This flow can cause a 20% variation in the atmospheric pressure on Mars. This was seen by the Viking landers.

Thermal Tides

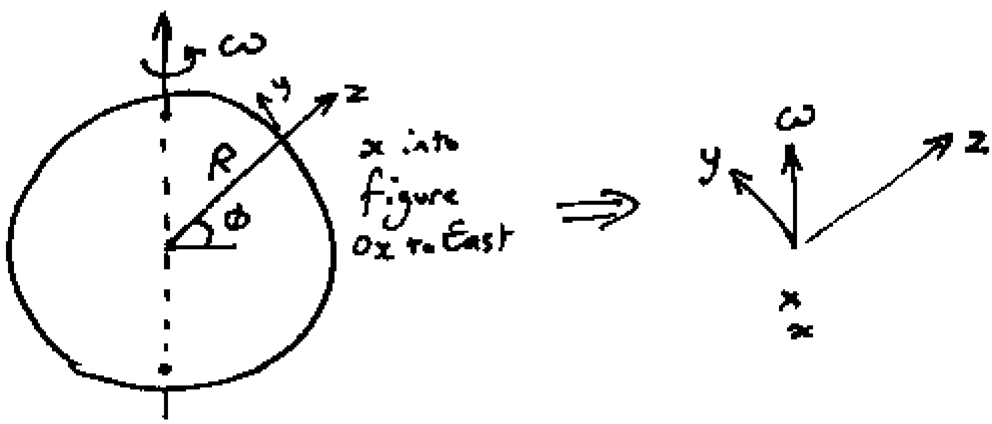
The subsolar point of heating moves across the planet as it rotates, following the Sun. This leads to diurnal pressure and temperature variations and harmonics - these are called Tides, diurnal, semi-diurnal, terdiurnal etc. This effect is strong for Mars (as heating and cooling of the surface is so rapid, and it is enhanced by dust absorption). Thermal tides are weak in the lower atmosphere of Ven the lower atmosphere of Venus but may have influenced the current rotation rate of the planet. It spins slowly in a retrograde direction (117 Earth days), and it has been suggested that there was an interaction between body tides (slowing the planet) and atmospheric tides (trying to speed it up) in Venus' early history. The thermal tides would build up as the planet slowed. The reason it has any rotation at all may be due to this.

Venus E-W winds

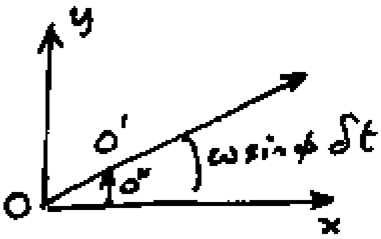
Venus has been found to have predominantly E-W winds. The Coriolis force on Venus us too small to cause this as it rotates so slowly, so it has been suggested these are the result of centrifugal rather than Coriolis forcing. The E-W winds are found to be about 1 ms^{-1} near the ground but rising to nearly 100 ms^{-1} near the cloud tops. They are global rather than "jet streams" which suggests the whole atmosphere "super-rotates". Hadley and eddy transport may "spread" the effect. In the high atmosphere there is also a 5-10 ms^{-1} pole to equator wind which shows large variations with time.

Coriolis Force and the Geostrophic Wind

If we look at the atmosphere in a frame fixed with respect to the Earth, we have in effect a rotating frame:



The motion of the x,y,z axes can be broken down into a circular motion (radius Rcos(phi)) with a centrifugal force and a rotationugal force and a rotational motion which gives the Coriolis force. We can give motions as components along the y and z axes. Note that angular rotations cannot be added as vectors: spins (rate of change of angle) however can. In the y-x plane:



In time δt parcel of air at O → O'' if no other forces acting

Let α be force to displace it from O'' to O' in time δt

$$O''O' = \frac{1}{2}\alpha(\delta t)^2$$

But $O''O' = OO''\,\omega \sin\phi\,\delta t$

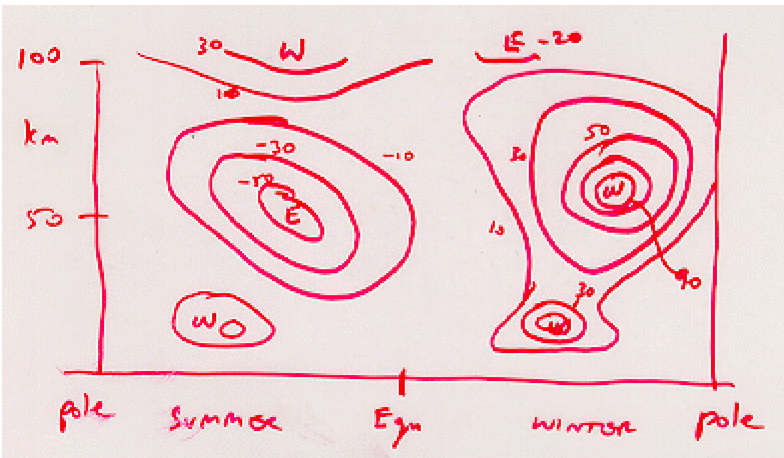
$= u\,\omega \sin\phi\,(\delta t)^2$

but $\alpha = -\frac{1}{\rho}\frac{dp}{dy}$ $\therefore$ $2u\omega\sin\phi = -\frac{1}{\rho}\frac{dp}{dy}$

The value u is called the **geostrophic wind**, and is parallel to the isobars. The more the wind is away from the isobars the more there is a time-dependent situation - i.w du/dt is not 0. In the northern hemisphere the wind rotates anti-clockwise around a low.

Mars too has a strong eastward-moving jet powered by baroclinic waves in the winter hemisphere close to the boundary of the seasonal CO_2 ice cap.

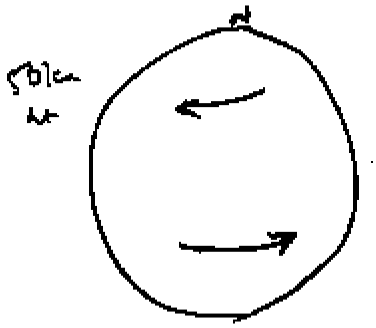
Earth's middle atmosphere average latitudinal wind structure:



This is dominated by Ozone heating. In the polar summer there is continuous input so there is a large summer-winter difference and a high-level wind pattern is set up.

Stratosphere and Lower Mesosphere

There is believed to be a fairly simple global wind pattern in the 20-50 km height range.



Looking at the Coriolis equation above, in summer northern hemisphere u <0 and phi > 0 so we expect dp/dy >0. In the winter hemisphere we get u >0 and phi < 0 so dp/dy > 0. Set u >0 and phi < 0 so dp/dy > 0. So, the pressure increases from winter to summer poles - a contract with ground level where the surface governs the heating, and we get maximum pressure from the maximum heating.

Coriolis wind in terms of T rather than phi

Note: this is not needed for the examination - I have included it just for completeness

Go back to pressure height formula

Take $g = g(z)$ (neglect latitude changes)

$$\ln \frac{p(y,z)}{p(y,z_0)} = -\int_{z_0}^{z} \frac{Mg}{RT(y,J)}\, dJ$$

J = height (dummy variable)

i.e.

$$\frac{1}{p(y,z)} \frac{\partial p(y,z)}{\partial y} - \frac{1}{p(y,z_0)} \frac{\partial p(y,z_0)}{\partial y} = -\int_{z_0}^{z} \frac{Mg}{R} \frac{\partial}{\partial y}\left(\frac{1}{T(y,J)}\right) dJ$$

Substitute & use $\frac{p}{\rho} = \frac{RT}{m}$

$$2\omega\sin\phi\, u \frac{M}{RT(y,z)} - \frac{1}{p(y,z_0)} \frac{\partial p(y,z_0)}{\partial y} = -\int_{z_0}^{z} \frac{Mg}{R} \frac{\partial}{\partial y}\left(\frac{1}{T(y,J)}\right) dJ$$

Differentiate w.r.t. z

$$2\omega\sin\phi \frac{\partial}{\partial z}\left(\frac{u}{T}\right) + 0 = g \frac{\partial}{\partial y}\left(\frac{1}{T}\right)$$

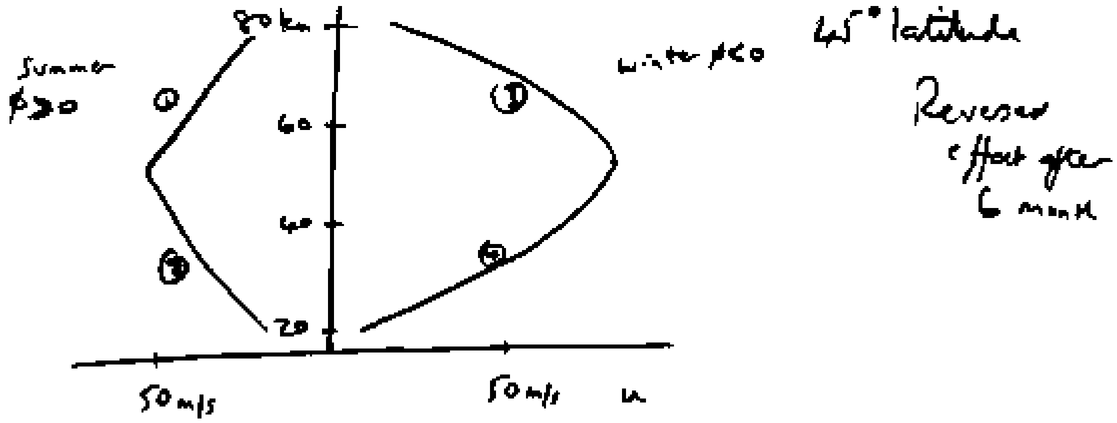
$$2\omega\sin\phi \left\{\frac{\partial}{\partial z}\left(\frac{u}{T}\right)\right\} = g \frac{\partial}{\partial y}\left(\frac{1}{T}\right)$$

partial as p, ρ etc all $f(z)$ as well as $f(y)$

This gives Coriolis wind in terms of T rather than p

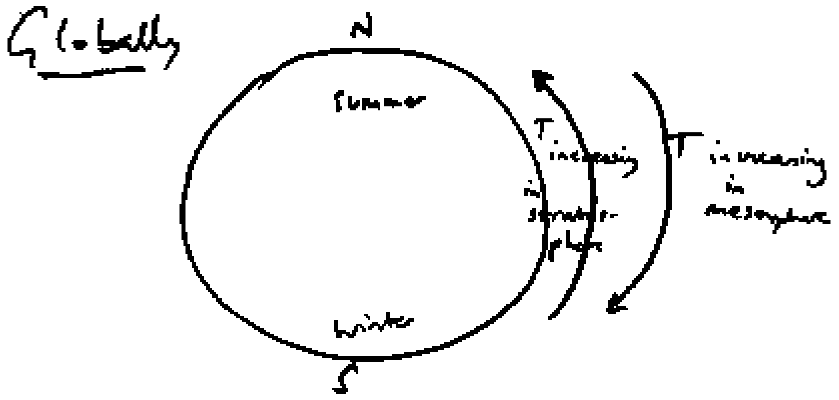
$$\Rightarrow \quad \frac{\partial T}{\partial y} = -\frac{2\omega\sin\phi}{g}\left[T\frac{\partial u}{\partial z} - u\frac{\partial T}{\partial z}\right]$$

2nd term small but probably not negligible



① $\frac{\partial T}{\partial z} < 0$ $\frac{du}{dz} > 0$ ③ $\frac{\partial u}{\partial z} < 0$ $\frac{\partial T}{\partial z} < 0$

② $\frac{\partial T}{\partial z} > 0$ $\frac{du}{dz} < 0$ ④ $\frac{du}{dz} > 0$ $\frac{\partial T}{\partial z} > 0$



What might be the explanation for this? The stratospheric temperature gradient is due to O_3 radiational heating. In the mesosphere adiabatic and chemical heating are possibilities. (Adiabatic: air approaches the winter pole, sinks, is compressed and hence heats up - this only needs cm/s descent speeds. Chemical: less likely but maybe $O + O \rightarrow O_2$. The amount of O is small 50-80 km).

One effect that is becoming more recognised as a major energy input and dynamic effect is the influence of Atmospheric Gravity waves (AGWs). These are generated by topography and weather systems low down, and also by ozone heating in the middle atmosphere, increase in amplitude with height and 'break' around the top of the mesosphere. Tides also die away in the mesopause/lower thermosphere region.

Martian Winds

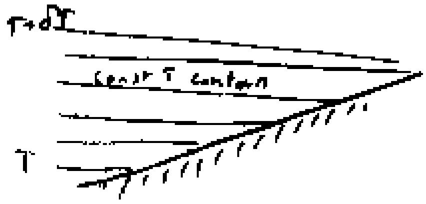
again this is non-examinable, and is only here for completeness

One possible explanation for Martian windle explanation for Martian winds is that they are caused by a sloping surface with a temperature gradient. We can look at the size of this effect for Earth by taking the equation:

$$\frac{\partial T}{\partial y} = -\frac{2\omega \sin\phi}{g}\left[T \frac{\partial u}{\partial z}\right]$$

Assume $\frac{\partial T}{\partial z} = 0$

For Earth omega=7 10^{-5} rad/s, g=9.81 m s^{-2}, T=250 K. Take a mid-latitude so sin(phi)=0.5



Assume (del-u/del-z) is about 1 m/s/km and (del-T/del-y) is 1.8/1000 K/km. Only a very small temperature gradient is needed.